# MTH401 MID TERM PAST PAPERS (FILE PART I) SOLVED

## BY MASOOM FAIRY

**Note:**

- **I could not make Neat File due to Much Load shedding.**
- **There is an other file because of Large size of this one.**

**FINALTERM EXAMINATION**

**SPRING 2007**

**MTH401 - DIFFERENTIAL EQUATIONS (Session - 4 )**

Question No: 1 ( Marks: 2 ) - Please choose one

The Wronskian of the function $W(e^{x}, e^{-x}) =$ ________ is

▶ 1

- 0
- **-2 LECTURE 13**
- None of these

Question No: 2 ( Marks: 2 ) - Please choose one

The eigen values of matrix $\begin{pmatrix} 2 & 3 \\ 2 & 1 \end{pmatrix}$ are

- $\lambda = 0, -1$
- $\lambda = 4, -1$
- $\lambda = 1, 5$ **According to Page 437**
- None of these

Question No: 3 ( Marks: 2 ) - Please choose one

Roots of the equation $y^{///} + y^{/} = 0$ will be

- ▶ 0,1,2
- ▶ 0,+ i , - i
- ▶ 0,1,i
- ▶ **None of these LECTURE 2 (SAPARIBLE EQUATIONS)**

Question No: 4 ( Marks: 2 ) - Please choose one

A differential equation $y^{//} + 16y = 0 \;\; with \;\; y(0) = 0 \;,\; y(\frac{\pi}{8}) = 0$ is called

- ▶ Initial value problem
- ▶ **Boundary value problem**

  **PAGE 108**
- ▶ None of these

Question No: 5 ( Marks: 2 ) - Please choose one

Suppose the functions $f_1(x), f_2(x), \ldots, f_n(x)$ possess at least n-1 derivatives on interval I , if $W(f_1, f_2, \ldots, f_n) \neq 0$ is called __________

- ▶ Linearly dependent
- ▶ **Linearly independent** **PAGE 110**
- ▶ None of these

| | MIDTERM EXAMINATION<br>SEMESTER FALL 2004<br>MTH401- Differential Equations | Total Marks: 50<br>Duration: 60min |
|---|---|---|

**Question No: 1** **Marks: 2**

Order of the differential equation is the highest order derivative in a differential equation.

**T**

F

**02**

$e^x$ Is integrating factor of the differential equation $y(x+y+1)dx+(x+2y)dy=0$.

T

F 02

**Question No: 3** **Marks: 2**

If $_1: G(x, y, c_1)=0$ orthogonal intersect all curves of another family $_2: H(x, y, c_2)=0$ then each curve of the families is said to be orthogonal trajectories of the other.

**T** **PAGE 74**

F 02

**Question No: 4** Marks:2

The given functions $f_1(x) = 1 + x,\ f_2(x) = x,\ f_2(x) = x^2$ are linearly independent.

**T**

F

02

**Question No: 5** Marks:

A set of functions whose wronskian is zero guarantees that set of functions is linearly dependent.

T

**F**

02

**(a)** Define separable form. Just separate the variables of the given differential equation. $(r\theta - 4r + \theta - 4)dr - \left(r^2\theta + 20r^2 - \theta - 20\right)d\theta = 0$

[redacted]

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form d y / d x = h(x)g(y)

$$(r\theta - 4r + \theta - 4)dr - \left(r^2\theta + 20r^2 - \theta - 20\right)d\theta = 0$$

$$(r\theta - 4r + \theta - 4)dr = \left(r^2\theta + 20r^2 - \theta - 20\right)d\theta$$

$$\left[r(\theta - 4) + 1(\theta - 4)\right]dr = r^2\left(\theta + 20\right) - 1(\theta + 20)\,d\theta$$

$$(r+1)(\theta - 4)dr = \left(r^2 - 1\right)(\theta + 20)d\theta$$

$$\frac{(r+1)}{(r^2-1)}dr = \frac{(\theta + 20)}{(\theta - 4)}d\theta$$

$$\frac{1}{r-1}dr = \frac{(\theta + 20)}{\theta - 4}d\theta$$

**(b)** Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$\left(2y^2 + 3x\right)dx + 2xydy = 0$$

[redacted]

$$(2y^2 + 3x)dx + 2xydy = 0$$
$$M(x,y) = 2y^2 + 3x,\ N(x,y) = 2xy$$
$$M_y = 4y,\ N_x = 2y$$
$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$(2y^2 + 3x)dx + 2xydy = 0$$
$$\frac{M_y - N_x}{N} = \frac{4y - 2y}{2xy} = \frac{2y}{2xy} = \frac{1}{x}$$
$$I.F = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$$
$$(2xy^2 + 3x^2)dx + 2x^2ydy = 0$$
$$M(x,y) = 2xy^2 + 3x^2,\ N(x,y) = 2x^2y$$
$$M_y = 4xy,\ N_x = 4xy$$
$$M_y = N_x$$

Which shows that now equation is exact

**Question No: 7** **Marks:10**

**(a)** Solve given Bernoulli equation $xy - \frac{dy}{dx} = y^3 e^{-x^2}$ **(Just make the given equation linear in v, do not integrate)**

**Solution**

$$xy - \frac{dy}{dx} = y^3 e^{-x^2} xy$$
$$-2 - \frac{dy}{dx} y^{-3} = e^{-x^2}$$
$$put\ y^{-2} = v$$
$$-2\frac{dy}{dx} y^{-3} =$$
$$\frac{dv}{dx} - \frac{dy}{dx} y^{-3}$$
$$= \frac{1}{2}\frac{dv}{dx}\ Then$$
$$\frac{1}{2}\frac{dv}{dx} + vx = e^{-x^2}$$

$\frac{dv}{dx} + 2vx = 2e^{-x^2}$

Thus it is linear in “v”.

| | MIDTERM EXAMINATION<br>SEMESTER FALL 2004<br>MTH401- Differential Equations | Total Marks: 50<br>Duration: 60min |
|---|---|---|

The differential equation $\frac{dy}{dx} - y = e^x y^2$ is not Bernoulli equation.

- T
- **F** PAGE 52

**Question No: 2** **Marks: 2**

$f(x, y) = \frac{x}{x_2 + y_2}$ Is homogeneous.

- T
- **F** PAGE 349

**Question No: 3** **Marks: 2**

Population dynamics are not practical application of the first order differential equations.

- T
- **F**

**Question No: 4** **Marks: 2**

A set

$$\{y_1, y_2, \dots, y_n\}$$

Of $n$ linearly dependent solutions, on interval $I$, of the homogeneous linear $n$th-order differential equation

$$a_n(x)\frac{d^n y}{dx^n} + a_{n-1}(x)\frac{d^{n-1} y}{dx^{n-1}} + \cdots + a_1(x)\frac{dy}{dx} + a_0(x)y = 0$$

Is said to be a fundamental set of solutions on the interval $I$.

- T
- **F**

**Question No: 5** **Marks: 2**

The differential operator that annihilates $10x^3\ 2x$ is $D^4$.

- **T** **PAGE 167**
- F

**Question No: 6** **Marks:10**

**(a)** Define separable form. Just separate the variables of the given differential equation.

$$(3r\theta - 3\theta + r - 1)dr - (2r\theta + 4\theta - r - 2)d\theta = 0$$

**Solution**

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form d y / d x = h(x)g(y)

1. $3r\theta - 3\theta + r - 1)dr - (2r\theta + 4\theta - r - 2)d\theta = 0$
2. $3r\theta - 3\theta + r - 1)dr = (2r\theta + 4\theta - r - 2)d\theta$

$$[3\theta(r - 1) + 1(r - 1)]dr = 2\theta(r + 2) - 1(r + 2)\ d\theta$$

$$(r - 1)(3\theta + 1)dr = (r + 2)(2\theta - 1)d\theta$$

$$\frac{(r - 1)}{(r + 2)}dr = \frac{(2\theta - 1)}{(3\theta + 1)}d\theta$$

**(b)** Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$$3y - \frac{2}{5}x^2 dy + \frac{x}{5} = 0$$

$y \quad dx \;\; 2y$

It can also be written as

$$xdx + 2\left(3y^2 - x^2\right)dy = 0$$

$$M(x, y) = x, \quad N(x, y) = 6y^2 - 2x^2$$

$$M_y = 0, \quad N_x = -4x$$

$$M_y \neq N_x$$

Thus it is not exact now we apply techniques to make it exact

$$xdx + \left(6y^2 - 2x^2\right)dy = 0$$

$$\frac{N_x - M_y}{M} = \frac{-4x}{x} = -4 = g(y)$$

$$I.F = e^{\int -4dy} = e^{-4y}$$

$$xe^{-4y}dx + e^{-4y}\left(3y^2 - 2x^2\right)dy = 0$$

$$M(x, y) = xe^{-4y}, \quad N(x, y) = e^{-4y}\left(3y^2 - 2x^2\right)$$

$$M_y = -4xe^{-4y}, N_x =$$

$$-4xe^{-4y} \; M_y = N_x$$

Which shows that equation is exact

**Question No: 7** **Marks:10**

**(a)** Solve the Bernoulli equation $x^3\frac{dy}{dx} + 2xy = y^5$ (**Just make the given equation linear in v, do not integrate**)

**Solution**

$$x^3\frac{dy}{dx} + 2xy = y^5$$

$$\frac{dy}{dx}y^{-5} + \frac{2}{x^2}y^{-4} = \frac{1}{x^3}$$

$$put \; y^{-4} = v$$

$$-4\frac{dy}{dx}y^{-5} = \frac{dv}{dx}$$

$$\frac{dy}{dx}y^{-5} = -\frac{1}{4}\frac{dv}{dx}$$

*Then*

$$-\frac{1}{4}\frac{dv}{dx} + \frac{2v}{x^2} = \frac{1}{x^3}$$

$$\frac{dv}{dx} - \frac{8v}{x^2} = -\frac{4}{x^3}$$

Thus it is linear in “v”.

| | MIDTERM EXAMINATION<br>SEMESTER FALL 2004<br>MTH401- Differential Equations | Total Marks: 50<br>Duration: 60min |
|---|---|---|

A differential equation said to be ordinary differential equation if it contains only ordinary derivatives with respect to single variable.

**T**

F PAGE 4

A solution of the differential equation of the form y= f (x) is called the implicit solution.

T

**F PAGE 6**

A relation ***G(x,y)*** is known as an implicit solution of a differential equation, if it defines one or more explicit solution on *I*.

Logistic equations are applications of non-linear equations.

**T**

F

**https://www.google.com.pk/url?sa=f&rct=j&url=http://en.wikipedia.org/wiki/Logistic_function&q=&esrc=s&ei=vnudUcjYOIej4gTB3ICYCQ&usg=AFQjCNFQfw3zqH_eBDWh5GGKpRpoSXlqUg**

The given functions $f_1(x)=5, f_2(x)=\cos^2 x, f_3(x)=\sin^2 x$ are linearly independent.

T

**F** **PAGE 112**

**Question No: 5** **Marks: 2**

A set of functions whose wronskian is zero then set of functions may or may not be dependent.

**T**

**F**

**http://math.berkeley.edu/~mannisto/lindiff2.pdf**

[redacted]

**(a)** Define separable form. Just separate the variables of the given differential equation.

$dy$

[redacted]

The differential equation of the form d y / d x = f (x, y) is called **separable** if it can be written in the form

d y / d x = h(x)g(y)

$\sec y \frac{dy}{dx} + \sin(x-y) = \sin(x+y)$

$\sec y \frac{dy}{dx} = \sin(x+y) - \sin(x-y)$

$\sec y \frac{dy}{dx} = \sin(x+y) - \sin(x-y)$

$\sec y \frac{dy}{dx} = \sin x \cos y + \cos x \sin y - \sin x \cos y + \cos x$

$\sin y \sec y \frac{dy}{dx} = 2\cos x \sin y$

$dy = \cos x dx$

$2\cos y \sin y \quad dy \quad \sin 2$

$y = \cos x dx$

$\cos ec2\, ydy = \cos xdx$

**(b)** Check whether the given differential equation is exact or not if not then make it exact also show that it is exact **(Just make the equation exact do not solve it further).**

$e^x dx + \left( e^x \cot y + 2y \cos ecy \right) dy = 0$

$e^x dx + \left(e^x \cot y + 2y \cos ecy\right)dy = 0$

$M(x,y) = e^x, \; N(x,y) = e^x \cot y + 2y \cos ecy$

$M_y = 0, \; N_x = e^x \cot y$

$M_y \neq N_x$

Thus it is not exact now we apply techniques to make it exact

$e^x dx + \left(e^x \cot y + 2y \cos ecy\right)dy = 0$

$$\frac{N_x - M_y}{M} = \frac{e^x \cot y - 0}{e^x} = \cot y$$

$I.F = e^{\int \cot y dy} = e^{\ln \sin y} = \sin y$

$e^x \sin y dx + \left(e^x \cos y + 2y\right)dy = 0$

$M(x,y) = e^x \sin y, \; N(x,y) = e^x \cos y + 2y$

$M_y = e^x \cos y, N_x = e^x \cos y$

$M_y = N_x$

Which shows that equation is exact

**Question No: 7** **Marks: 10**

**(a)** Solve Bernoulli equation $y + 2\frac{dy}{dx} = y^3(x-1)$ (**Just make the given equation linear in v, do not integrate**)

**Solution**

$xy - \frac{dy}{dx} = y^3(x-1)$

$xy^{-2} - \frac{dy}{dx} y^{-3} = (x-1)$

$put \; y^{-2} = v$

$-2\frac{dy}{dx} y^{-3} = \frac{dv}{dx}$

$-\frac{dy}{dx} y^{-3} = \frac{1}{2}\frac{dv}{dx}$

$Then$

$\frac{1}{2}\frac{dv}{dx} + vx = (x-1)$

$\frac{dv}{dx} + 2vx = 2(x-1)$

**(b)** The radioactive isotope of the lead, Pb-209, decay at a rate proportional to the amount present at any time and has a half-life of 4 hours. If 2 grams of the lead is present initially, how long will it take for 80% of the lead to decay? **(Just make the model of the radioactive decay as well as describe the given conditions do not solve further)**

# MTH401 MID TERM PAST PAPERS (FILE PART I) SOLVED

# BY MASOOM FAIRY

## *Note:*

- **I could not make *Neat* File, due to Much Load shedding.**
- **There is an other file because of Large size of this one.**