FINALTERM EXAMINATION
Fall 2009

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

Let $f(x)$ is a function such that as x approaches a real number a, either from left or right-hand-side, the function values increases or decreases unboundedly then

$\lim_{x \to a} f(x)$

- ▶ Exist
- ▶ Does not exist

Question No: 2 (Marks: 1) - Please choose one

$\frac{d(\sec x)}{dx} =$

- ▶ $(\sec x)(\tan x)$
- ▶ $(\sec x)(\tan x)$
- ▶ $(co\sec x)(co\,t\,x)$
- ▶ $(co\sec x)(\tan x)$

Question No: 3 (Marks: 1) - Please choose one

Consider a function $h(x)$ and a constant c then

$\frac{d}{dx}((c)\{h(x)\}) =$ _____

- ▶ 0

- $\frac{d}{dx}(h(x))$
- $\frac{d}{dx}(h(cx))$
- $c\frac{d}{dx}(h(x))$

Question No: 4 (Marks: 1) - Please choose one

If f is continuous function such that $\lim_{x\to-\infty} f(x) = +\infty$ *and* $\lim_{x\to+\infty} f(x) = +\infty$ then f has _____ on $(-\infty, +\infty)$

- maximum value but no minimum
- minimum value but no maximum
- both maximum and minimum value

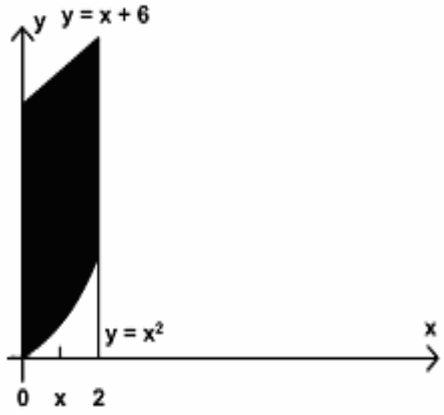
Question No: 5 (Marks: 1) - Please choose one

Sigma notation is represented by which of the following Greek letter?

- χ
- η
- Σ
- ψ

Question No: 6 (Marks: 1) - Please choose one

In the following figure, the area enclosed is bounded below by **:**



- $y = x + 6$
- $y = x^2$
- $x = 2$
- $x = 0$

Question No: 7 (Marks: 1) - Please choose one

At what points the two curves: $y = x^2 \ and \ y = x + 6$ intersect ?

- $x = 0 \ and \ x = 2$
- $x = 0 \ and \ x = 3$
- $x = 2 \ and \ x = 3$
- $x = -2 \ and \ x = 3$

Question No: 8 (Marks: 1) - Please choose one

Let the solid generated by the region enclosed between $y = \sqrt{x} \quad ; \quad x = 1, x = 4$ and the x-axis is revolved about the y-axis. Which of the following equation gives the volumes of a solid by cylindrical shells?

- $V = \int_1^4 2\pi x\sqrt{x}dx$
- $V = \int_1^4 2x\sqrt{x}dx$
- $V = \int_0^4 2x\sqrt{x}dx$
- $V = \int_{-4}^4 2x\sqrt{x}dx$

Question No: 9 (Marks: 1) - Please choose one

Let f is a smooth curve on the interval [a, b]. What is the arc length L of the curve f(x) defined over the interval [a, b]?

$$L = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))}$$

▶

$$L = \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))} \Delta x_k$$

▶

$$L = \lim_{\max \Delta x \to 0} \sum_{k=1}^{n} \sqrt{1 + (f'(x^*_k))^2} \Delta x_k$$

▶

$$L = \sum_{k=1}^{n} \sqrt{1 + (f(x^*_k))} \Delta x$$

▶

Question No: 10 (Marks: 1) - Please choose one

For a graph to be symmetric about y-axis means, for each point (x,y) on the graph,the point ---------- is also on the graph

▶ (x , -y)
▶ (-x , y)
▶ (-x , -y)

Question No: 11 (Marks: 1) - Please choose one

The graph $x = y^2$ is symmetric about ---------axis

▶ X-axis
▶ Y-axis
▶ Origin

Question No: 12 (Marks: 1) - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x determines exactly one value of y , we say that y is ……. of x

▶ relation

- ▶ function
- ▶ not a function
- ▶ not a relation

Question No: 13 (Marks: 1) - Please choose one

Domain of the function $y = \frac{(x^2-4)}{(x-2)}$ is

- ▶ $(-\infty, 2) U (2, +\infty)$
- ▶ $(-\infty, 2)$
- ▶ $(-\infty, 0)$

Question No: 14 (Marks: 1) - Please choose one

Tan(x) is continuous every where except at points

- ▶ $\pm\frac{k\pi}{2}(k = 1,3,5,...)$
- ▶ $\pm\frac{k\pi}{2}(k = 2,4,6,...)$
- ▶ $\pm\frac{k\pi}{2}(k = 1,2,3,4,5,6,...)$

Question No: 15 (Marks: 1) - Please choose one

$\lim_{x\to 0}\frac{\sin x}{x}$ = ----------

- ▶ -1
- ▶ 2
- ▶ 0
- ▶ 1

Question No: 16 (Marks: 1) - Please choose one

How the series $1-3+5-7+9-11$ can be expressed in sigma notation?

$\sum_{k=0}^{k=5}(-1)^k(2k+1)$

▶

$\sum_{k=1}^{k=5}(-1)^k(2k+1)$

▶

$\sum_{k=1}^{k=5}(2k+1)$

▶

$\sum_{k=1}^{k=5}(2k+1)$

▶

Question No: 17 (Marks: 1) - Please choose one

Let the region bounded by the curve $y=\sqrt[3]{x}$, the x-axis, and the line $x=3$ is revolved about the y-axis to generate a solid. Which of the following equation gives the volume of a solid by cylindrical shells?

$V=\int_0^3 x^{\frac{3}{2}}\,dx$

▶

$V=2\pi\int_0^3 \sqrt{x}\,dx$

▶

$V=\int_0^3 2\pi x\sqrt[3]{x}\,dx$

▶

$V=\int_0^3 x\sqrt[3]{x}\,dx$

▶

Question No: 18 (Marks: 1) - Please choose one

$$y = \frac{2\sqrt{2}}{3}x^{\frac{3}{2}} \; ; \; 0 \le x \le 2$$

Let then which of the following is the length of the curve?

$$L = \int_0^2 \sqrt{\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\, dx$$

▶

$$L = \int \sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\, dx$$

▶

$$L = \int_0^2 \sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]^2}\, dx$$

▶

$$L = \int_0^2 \sqrt{1+\left[\frac{d}{dx}\left(\frac{2\sqrt{2}}{3}x^{\frac{3}{2}}\right)\right]}\, dx$$

▶

Question No: 19 (Marks: 1) - Please choose one

$\frac{2}{3}$ is known as

▶ An even number
▶ Irrational Number
▶ A natural Number

▶ Rational Number

Question No: 20 (Marks: 1) - Please choose one

For a function *f*, let $f'(x_n) = 0 \quad for\ some\ n$.

Does the Newton's Method works for approximating the solution of $f(x) = 0$?

▶ Yes
▶ No

Question No: 21 (Marks: 1) - Please choose one

The Mean Value Theorem states that "Let function f be differentiable on (a,b) and continuous on [a, b], then there exist at least one point c in (a,b) where ……. "

▶ $f'(c) = \frac{f(b) - f(a)}{b - a}$

$f(c) = \frac{f(b) - f(a)}{b - a}$

▶

$f(c) = \frac{f(a) - f(b)}{b - a}$

▶

$f'(c) = \frac{f(a) - f(b)}{b - a}$

▶

Question No: 22 (Marks: 1) - Please choose one

$$\frac{d}{dx}[F(x)] = f(x)$$

If there is some function F such that then any function of the form $F(x) + C$ is ---------- of $f(x)$

▶ Derivative
▶ Antiderivative
▶ Slope
▶ Maximum value

Question No: 23 (Marks: 1) - Please choose one

If f and g are continues function on an interval [a, b] and $f(x) \geq g(x) \ \ for \ \ a \leq x \leq b$, then area is bounded by the lines parallel to:

▶ X -axis
▶ Y-axis
▶ Both X -axis and Y-axis

Question No: 24 (Marks: 1) - Please choose one

What is the sum of following series?

$1^3 + 2^3 + 3^3 + 4^3 + \_\_\_\_\_ + n^3$

$\frac{n(2n)(2n+1)}{6}$

▶

$\frac{(n+1)(n+2)}{2}$

▶

$\left[\frac{n(n+2)}{2}\right]^2$

▶

$\left[\frac{n(n+1)}{2}\right]^2$

▶

Question No: 25 (Marks: 1) - Please choose one

$\frac{5}{7}\times 1^2+\frac{5}{7}\times 2^2+\frac{5}{7}\times 3^2+\frac{5}{7}\times 4^2....+\frac{5}{7}\times n^2 =$ _____

$\frac{5n(n+1)(2n+1)}{42}$

▶

$\frac{5n(n+1)}{14}$

▶

$\frac{5n^2(n+1)^2}{14}$

▶

$\frac{5(n+1)(2n+1)}{42}$

▶

Question No: 26 (Marks: 1) - Please choose one

$\int_a^a f(x)dx =$ _____

If point a is in the domain of function f, then

▶ $f'(x)$

▶ $f(x)$

► 0
► 1

Question No: 27 (Marks: 1) - Please choose one

If $a_1 > a_2 > a_3 > ..... > a_n > ....$, then a sequence $\{a_n\}$ is .....

► Increasing
► Nondecreasing
► Decreasing
► Nonincreasing

Question No: 28 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the difference between successive terms $a_{n+1} - a_n \leq 0$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 29 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} < 1$ then the sequence is known as**:**

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 30 (Marks: 1) - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\frac{a_{n+1}}{a_n} \geq 1$ then the sequence is known as **:**

► Increasing
► Decreasing
► Nondecreasing

► Nonincreasing

Question No: 31 (Marks: 1) - Please choose one

$$a_n = \left\{\frac{1}{n}\right\}_{n=1}^{\infty}$$

Which of the following option is true for the sequence ?

► Increasing
► Decreasing
► Nonincreasing
► Nondecreasing

Question No: 32 (Marks: 1) - Please choose one

If the partial sum of a series is finite then the series will/will be**:**

► Divergent
► Convergent
► Give no information

Question No: 33 (Marks: 1) - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ... \ \ where\ (a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

► Converges
► Diverges
► Gives no information

Question No: 34 (Marks: 1) - Please choose one

$$\rho = \lim_{k \to +\infty} \frac{u_{k+1}}{u_k}$$

If where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

► Convergent
► Divergent
► Give no information

Question No: 35 (Marks: 1) - Please choose one

$$\rho = \lim_{k \to +\infty} \sqrt[k]{u_k}$$

If where $\rho > 1$ then the series $\sum u_k$ with positive terms will /will be.....?

► Convergent

► Divergent
► Give no information

Question No: 36 (Marks: 1) - Please choose one

In alternating series test, which one of the following condition must be satisfied?

► $\lim_{k\to\infty} a_k = 1$

► $a_1 > a_2 > a_3 ..... > a_k > ....$

► $a_1 \le a_2 \le a_3 ..... \le a_k \le ....$

Question No: 37 (Marks: 1) - Please choose one

$$\sum_{k=1}^{\infty} (-1)^n a_k$$

A series of the form is called _____.

► Alternating series
► Geometric series
► Arithmetic series
► Harmonic series

Question No: 38 (Marks: 1) - Please choose one

Which of the following is the Maclaurin series for e^x ?

► $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^k}{k!} + ...$

► $x + \frac{x^2}{2!} + \frac{x^3}{3!} + ... + \frac{x^k}{k!} + ...$

► $1 + x + \frac{x^3}{3!} + ... + \frac{x^k}{k!} + ...$

► $1 - x + \frac{x^3}{3!} - ... - \frac{x^k}{k!} - ...$

Question No: 39 (Marks: 1) - Please choose one

Which of the following is the work done *W* if an object moves in the positive direction along a coordinate line while subject to a force *F(x)* in the direction of motion over an interval [0,3]?

$$W = \int_{2}^{3} 3x dx$$

▶

$$W = \int_{0}^{3} 3x dx$$

▶

$$W = \int_{0}^{3} F(x) dx$$

▶

$$W = \int_{3}^{0} F(x) dx$$

▶

Question No: 40 (Marks: 1) - Please choose one

Which of the following is the spring constant *k* if a spring whose natural length is 2*m* exerts a force of 3*N* when stretched 1*m* beyond its natural length?

▶ 3 *x*
▶ 3 *N/m*
▶ 2 *m*
▶ 3 *m/N*

Question No: 41 (Marks: 2)

Evaluate the following integral by substitution method.

$$\int x\,(2x^2+1)^{\frac{2}{3}}\,dx$$

Question No: 42 (Marks: 2)

Find the limits of the integral indicating the area bounded by the curves $y = x^2 \text{ and } y = x + 6$.

Sol,

Question No: 43 (Marks: 2)

What will be the amount of work done if an object moves 7*m* in the direction of a force of 70*N*?

Question No: 44 (Marks: 3)

Evaluate the following integral:

$$\int \frac{5-6\sin^2 x}{\sin^2 x}\, dx$$

Question No: 45 (Marks: 3)

Find a definite integral indicating the area of the surface generated by revolving the curve $y = \sqrt[3]{3x} \;;\quad 0 \le y \le 4$ about the *x* – axis. But do not evaluate the integral.

Question No: 46 (Marks: 3)

Find the spring constant '*k*'; if a force of 10*N* is required to stretch a spring from its natural length of 4.8*m* to a length of 6.8*m*?

Question No: 47 (Marks: 5)

Let $\frac{d}{dx}[f(x)] = 12x^2 - 6x + 1$. Find $f(x)$

Sol,

Question No: 48 (Marks: 5)

Use the cylindrical shell to find the volume of the solid generated when the region enclosed by the curve $y = x^3 \;, x = 1,\; y = 0$ is revolved about the *y*-axis.

Question No: 49 (Marks: 5)

Determine whether the sequence $\{a_n\}$ converges or diverges; if it converges then find its limit;

$$a_n = \frac{3n^4 + 1}{4n^2 - 1}$$

where

Question No: 50 (Marks: 10)

Find the area of the region that is enclosed by the curves $y = x^2$ and $y = \sqrt{x}$ between $x = \frac{1}{4}$ and $x = 1$.