FINALTERM EXAMINATION 2009
(Session - 2)
<u>Calculus & Analytical Geometry-I</u>

Question No: 1   (Marks: 1)   - Please choose one

According to Power-Rule of differentiation, if $f(x) = x^n$ where n is a real number, then

$$\frac{d}{dx}[x^n] =$$

▶ x^{n-1}

▶ $n\,x^{n-1}$

▶ $n\,x^{n+1}$

▶ $(n-1)x^{n+1}$

Question No: 2   (Marks: 1)   - Please choose one

If a function g is differentiable at a point x and a function f is differentiable at a point g(x), then the _____ is differentiable at point x .

▶ Composition   (f o g)
▶ Quotient   (f / g)
▶ Product   (f . g)

▶ Sum   (f + g)

Question No: 3   (Marks: 1)   - Please choose one

If $y = f\left(g\left(h(x)\right)\right)$

$u = g\left(h(x)\right)$

$v = h(x)$ then $\dfrac{dy}{dx} = $ _____

▶ $\dfrac{dy}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dv}{dx}$

▶ $\dfrac{dy}{du} \dfrac{du}{dv} \dfrac{dv}{dx}$

▶ $\dfrac{dv}{du} \cdot \dfrac{du}{dv} \cdot \dfrac{dy}{dx}$

Question No: 4   (Marks: 1)   - Please choose one

If a function f is ……….. on a closed interval [a,b] , then f has both maximum and minimum value on [a,b].
▶ Continuous
▶ Discontinuous
▶ None of these

Question No: 5   (Marks: 1)   - Please choose one

$$\int_{a}^{x} \frac{t^2}{2} dt$$

The expression   , represents a function of :

▶ t
▶ a
▶ Both x and a
▶ x

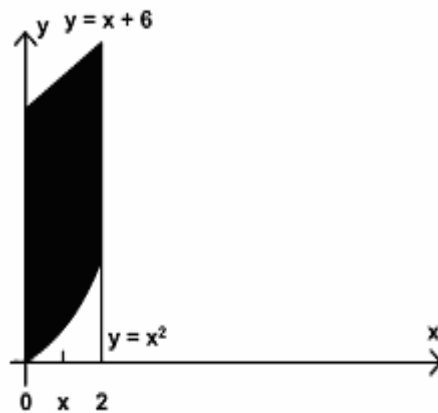**Question No: 6   (Marks: 1)   - Please choose one**

$$\int cf(x)dx = \text{_____}$$

if c is a constant

▶ 0

▶ c

$\int f(cx)dx$

▶

<mark>$c\int f(x)dx$</mark>

▶

Question No: 7   (Marks: 1)   - Please choose one

In the following figure, the area enclosed is bounded below by :

▶ $y = x + 6$

▶ <mark>$y = x^2$</mark>

▶ $x = 2$

▶ $x = 0$

Question No: 8   (Marks: 1)   - Please choose one

At what points the two curves: $y = x^2 \ and \ y = x + 6$ intersect ?

▶ $x = 0 \ and \ x = 2$

▶ $x = 0 \ and \ x = 3$

▶ $x = 2 \ and \ x = 3$

▶ <mark>$x = -2 \ and \ x = 3$</mark>

Question No: 9   (Marks: 1)   - Please choose one

What is the sum of following series?

$1 + 2 + 3 + 4 + _____ + n$

► $\dfrac{n+1}{2}$

► $\dfrac{(n+1)(n+2)}{2}$

► $\dfrac{n(n+2)}{2}$

► $\dfrac{n(n+1)}{2}$

Question No: 10   (Marks: 1)   - Please choose one

$If \; b > 0 \; then \; \dfrac{d}{dx}[b^x] = _____$

► 0

► xb^{x-1}

► $\ln b$

► $b^x \; \ln b$

Question No: 11   (Marks: 1)   - Please choose one

Let S be a solid bounded by two parallel planes perpendicular to the x-axis at x = a and x = b .If , for each x in [a,b], the cross-section area of S perpendicular to the x-axis is A(x) ,then what is the volume of the solid?

► $V = \displaystyle\int_a^b A(y)dx$

► $V = \displaystyle\int_a^b A(x)dx$

$$V = \int_0^{A(x)} [b - a]\,dx$$

▶

$$V = \int_0^{A(x)} [b + a]\,dx$$

▶

Question No: 12   (Marks: 1)   - Please choose one

Let the solid generated by the region enclosed between

$y = \sqrt{x} \quad ; \quad x = 1, x = 4$

and the x-axis is revolved about the y-axis. Which of
the following equation gives the volumes of a solid by cylindrical shells?

$$V = \int_1^4 2\pi x \sqrt{x}\,dx$$

▶

$$V = \int_1^4 2x \sqrt{x}\,dx$$

▶

$$V = \int_0^4 2x \sqrt{x}\,dx$$

▶

$$V = \int_{-4}^4 2x \sqrt{x}\,dx$$

▶

Question No: 13   (Marks: 1)   - Please choose one

If slope m of a none vertical line is m =1 then the angle of inclination of the line
is

$\dfrac{\pi}{4}$

▶

$$\frac{\pi}{2}$$

►

$$\frac{\pi}{5}$$

►

Question No: 14   (Marks: 1)   - Please choose one

The PYTHAGORAS theorem describes the relationship between the sides of
…………………..

- ► Right angle triangle
- ► Isoceleous triangle
- ► Equilateral triangle

Question No: 15   (Marks: 1)   - Please choose one

If a quantity y depends on another quantity x in such a way that each value of x
determines exactly one value of y , we say that y is ………………. of x

- ► relation
- ► function
- ► not a function
- ► not a relation

Question No: 16   (Marks: 1)   - Please choose one

The graph of the equation $y = x^2 - 4x + 5$ will represent

- ► Parabola
- ► Straight line
- ► Two straight lines

- ► Ellipse

Question No: 17   (Marks: 1)   - Please choose one

$$\lim_{x \to a} f(x) = ...where\ f(x) = k$$

The   (k is a constant)

► k+2
► k+1
► **k**

Question No: 18   (Marks: 1)   - Please choose one

Consider the indefinite integral
$$\int \frac{1}{x^2+2} \, dx$$

Let $t = x^2 + 2$

Is the following substitution correct?
$$\int \frac{1}{x^2+2} \, dx = \int \frac{1}{t} \, dt$$

► **Yes**
► No

Question No: 19   (Marks: 1)   - Please choose one

$$\log_b \frac{1}{t} = \underline{\qquad}$$

► $\log_b t$

► $1 - \log_b t$

► $1 + \log_b t$

► **$-\log_b t$**

Question No: 20   (Marks: 1)   - Please choose one

How the series $1 - 3 + 5 - 7 + 9 - 11$ can be expressed in sigma notation?

► **$\sum_{k=0}^{k=5} (-1)^k (2k+1)$**

►

$$\sum_{k=1}^{k=5}(-1)^k(2k+1)$$

►

$$\sum_{k=1}^{k=5}(2k+1)$$

►

$$\sum_{k=1}^{k=5}(2k+1)$$

►

Question No: 21   (Marks: 1)   - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\,\Delta x_k$$

The sum   is known as:

► Riemann Sum
► General Sum
► Integral Sum
► Geometric Sum

Question No: 22   (Marks: 1)   - Please choose one

$$\sum_{k=1}^{n} f(x^*_k)\Delta x_k$$

What does 'n' represent in the Riemann Sum   ?
► No. of Circles
► No. of Subintervals
► No. of Loops
► No. of Squares

Question No: 23   (Marks: 1)   - Please choose one

If w and v are continuous functions of y on an interval [c, d]
and $w(y) \geq v(y)\ for\ c \leq y \leq d$, then area is bounded by the lines parallel to: **:**
► Y-axis
► X-axis
► Both X-axis and Y-axis

Question No: 24   (Marks: 1)   - Please choose one

How the series $2(1) + 2(2) + 2(3) + 2(4) + 2(5)$ can be expressed in sigma notation?

$$\sum_{k=0}^{5} 2k^2$$

▶

$$\sum_{k=1}^{5} 2k^2$$

▶

$$\sum_{k=0}^{5} 2k$$

▶

$$\sum_{k=1}^{5} 2k$$

▶

Question No: 25   (Marks: 1)   - Please choose one

$$\sum_{k=1}^{n} \frac{k^3}{2} = \underline{\hspace{2cm}}$$

$$\frac{n(n+1)}{4}$$

▶

$$\frac{[n(n+1)]^2}{8}$$

▶

$$\frac{n(n+1)(2n+1)}{12}$$

▶

$$\frac{(n+1)(2n+1)}{6}$$

▶

Question No: 26   (Marks: 1)   - Please choose one

If $a_1 < a_2 < a_3 < ..... < a_n < ....$, then a sequence $\{a_n\}$ is.........

▶ Nondecreasing
▶ Decreasing

► Increasing
► Nonincreasing

Question No: 27   (Marks: 1)   - Please choose one

If $a_1 \geq a_2 \geq a_3 \geq ..... \geq a_n \geq ....$, then a sequence $\{a_n\}$ is ....

► Increasing
► Nondecreasing
► Decreasing
► Nonincreasing

Question No: 28   (Marks: 1)   - Please choose one

If the difference between successive terms $a_{n+1} - a_n > 0$ then the sequence $\{a_n\}$ is known as:

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 29   (Marks: 1)   - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\dfrac{a_{n+1}}{a_n} > 1$ then the sequence is known as:

► Increasing
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 30   (Marks: 1)   - Please choose one

For a sequence $\{a_n\}$ if the ratio of successive terms $\dfrac{a_{n+1}}{a_n} \geq 1$ then the sequence is known as :

► Increasing
► Decreasing
► Nondecreasing

► Nonincreasing

Question No: 31   (Marks: 1)   - Please choose one

If $f(n) = a_n$ is the nth term of the sequence and $f(n)$ is differentiable and $f'(n) > 0$ then the sequence will be:

► **Increasing**
► Decreasing
► Nondecreasing
► Nonincreasing

Question No: 32   (Marks: 1)   - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ...$ $where\ (a \neq 0)$, $|r| < 1$ then which of the following is true for the given series?

► **Converges**
► Diverges
► Gives no information

Question No: 33   (Marks: 1)   - Please choose one

If the geometric series $a + ar + ar^2 + ar^3 + ... + ar^{k-1} + ...$ $where\ (a \neq 0)$ $|r| \geq 1$, then which of the following is true for the given series?

► Converges
► **Diverges**
► Gives no information

Question No: 34   (Marks: 1)   - Please choose one

If the series $\sum_{k=1}^{\infty} u_k$ converges but the series $\sum_{k=1}^{\infty} |u_k|$ does not converge , then $\sum_{k=1}^{\infty} u_k$ will _____

► **Converge absolutely**
► Diverge
► Converge conditionally

Question No: 35   (Marks: 1)   - Please choose one

Let $\sum u_k$ be a series with nonzero terms and suppose that

$$\rho = \lim_{k \to \infty} \frac{|u_{k+1}|}{|u_k|} > 1$$

then which of the following is true?

► The series $\sum |u_k|$ diverges
► No conclusion can be drawn.

Question No: 36   (Marks: 1)   - Please choose one

Suppose f and g are integrable functions on $[a, b]$ and c is a constant, then

$$\int_a^b c\,[f(x) + g(x)]\, dx = \underline{\hspace{3cm}}$$

►
$$\int_a^b f(cx)dx + \int_a^b g(cx)dx$$

►
$$\int_a^b f(x)\, dx + \int_a^b g(x)]\, dx$$

►
$$c\int_a^b f(x)dx + c\int_a^b g(x)dx$$

► 0

Question No: 37   (Marks: 1)   - Please choose one

Which of the following is surface area S generated by revolving the curve $y = f(x)$ between
$x = 0$ and $x = 2$ about the x-axis?

$$S = \int_0^2 2\pi f(x)\sqrt{1+[f(x)]}\,dx$$

▶

▶

$$S = \int_0^2 2\pi f(x)\sqrt{1+[f'(x)]}\,dx$$

▶

$$S = \int_0^2 2\sqrt{1+[f'(x)]}\,dx$$

▶

Question No: 38   (Marks: 1)   - Please choose one

Which of the following is area of the surface generated by revolving the curve $y = 4\sqrt{x}\ ;\ 1 \le x \le 4$ about the x-axis?

$$\int_1^4 2\pi\left(4\sqrt{x}\right)\sqrt{1+\left[\left(4\sqrt{x}\right)\right]^2}\,dx$$

▶

$$\int_1^4 2\pi\left(4\sqrt{x}\right)\sqrt{1+\left[\left(4\sqrt{x}\right)'\right]}\,dx$$

▶

$$\int_1^4 2\pi+\sqrt{1+\left[\left(4\sqrt{x}\right)'\right]^2}\,dx$$

▶

$$\int_1^4 2\pi\left(4\sqrt{x}\right)\sqrt{1+\left[\left(4\sqrt{x}\right)'\right]^2}\,dx$$

▶

Question No: 39   (Marks: 1)   - Please choose one

Which of the following is the work done W if an object moves in the positive direction along a coordinate line while subject to a force $F(x)$ in the direction of motion over an interval [0,3]?

$$W = \int_2^3 3x\,dx$$

►

$$W = \int_0^3 3x\,dx$$

►

$$W = \int_0^3 F(x)\,dx$$

►

$$W = \int_3^0 F(x)\,dx$$

►

Question No: 40   (Marks: 1)   - Please choose one

$$\int_1^0 f(x)\,dx = 2 \qquad \int_0^5 f(x)\,dx = 1$$

If   and   then which of the following is value of

$$\int_1^5 f(x)\,dx$$

?
► -3
► -1
► 1
► 3

Question No: 41   (Marks: 2)

$$\frac{1}{2\sqrt{x}}$$

Derivative of a function is   . Find the original function.

Question No: 42   (Marks: 2)

$$\rho = \lim_{k \to +\infty} \sqrt[k]{u_k}$$

If $\sum u_k$ is a series with positive terms and   ; then write the three cases for the series to be convergent, divergent or none.

Question No: 43   (Marks: 2)

Evaluate the following integral:

$$\int_{1}^{3} \frac{1}{x^2}\, dx$$

Question No: 44   (Marks: 3)
Use the first fundamental theorem of calculus to evaluate the definite integral:

$$\int_{0}^{2} f(x)\, dx \quad where \quad f(x) = \begin{cases} x^2 & ; \quad 0 \le x < 1 \\ x^3 & ; \quad 1 \le x \le 2 \end{cases}$$

Question No: 45   (Marks: 3)

$$\sum_{k=2}^{\infty} (-1)^{k-1} \frac{2^{k-1}}{(k-1)!}$$

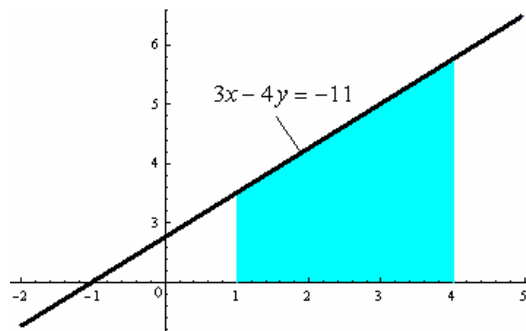Show that the series   converges absolutely.

Question No: 46   (Marks: 3)
Express the following definite integral as limit of Riemann Sum. (Do not evaluate the integral)

$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} Cos\, x \, dx$$

Question No: 47   (Marks: 5)
Express area of the shaded region as a definite integral.

$$3x - 4y = -11$$

Question No: 48   (Marks: 5)
How much work is required to wind the chain onto the pulley if a 100-ft length of steel chain weighting
15 lb/ ft. is dangling from a pulley?

Question No: 49   (Marks: 5)

Evaluate the following integral:

$$\int_1^2 \frac{x^2 + \sqrt{x}}{x^2} \, dx$$

Question No: 50   (Marks: 10)

Use L'Hopital's Rule to evaluate

$$\lim_{x \to 0} (1 + \sin x)^{\cot x}$$