FINALTERM EXAMINATION
Fall 2008
(Session - 1)

Calculus & Analytical Geometry-I

Question No: 1 (Marks: 1) - Please choose one

If $y = f(x)$ then the average rate of change of y with respect to x over the interval $[x_0, x_1]$ is the ………………………… Joining the points $(x_0, f(x_0)) and (x_1, f(x_1))$ on the graph of f

► Slope of the secant line
► Slope of tangent line
► Secant line
► none of these

Question No: 2 (Marks: 1) - Please choose one

Natural domain of $\frac{(x^2-4)}{(x-2)}$ is

► $(-\infty, 2) U (2, +\infty)$
► $(-\infty, 2)$
► $(-\infty, 0)$
► None of these

Question No: 3 (Marks: 1) - Please choose one

The equation $(x+4)^2+(y-1)^2=6$ represents a circle having center at ……….. and radius ....

- ► $(-4,1)\,,\sqrt{6}$
- ► $(-4,1)\,,6$
- ► $(-4,-1)\,,\sqrt{6}$
- ► None of these

Question No: 4 (Marks: 1) - Please choose one

The series $\sum u_k$ be a series with positive terms and suppose that $\rho=\lim_{k\to\infty}\sqrt[k]{u_k}=\lim_{k\to\infty}(u_k)^{\frac{1}{k}}$ if $\rho>1$ then the series

- ► Converges
- ► **Diverges**
- ► May converge or diverge
- ► None of these

Question No: 5 (Marks: 1) - Please choose one

The series $\sum u_k \; and \; \sum v_k$ are convergent series then $(\sum u_k+\sum v_k)\; and \;(\sum u_k-\sum v_k)$ will be ....and................

- ► Convergent, convergent
- ► Divergent, divergent
- ► **Convergent, divergent**
- ► Divergent, convergent

Question No: 6 (Marks: 1) - Please choose one

The notation $\{\frac{1}{2^n}\}_{-1}^{n}$ represents the sequence

► $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

► $0, 1, 2, 3 \ldots$

► $0, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

► None of these

Question No: 7 (Marks: 1) - Please choose one

If f is continuous on (a,b] but does not have a limit from the right then the integral defined by

$$\int_a^b f(x)dx = \lim_{l \to a} \int_l^b f(x)dx$$

is called .... Integral

► Improper
► Proper
► None of these

Question No: 8 (Marks: 1) - Please choose one

An object is displaced 1m by a force of 1N then the work done W is

► 2
► 0
► None of these
► **1**

Question No: 9 (Marks: 1) - Please choose one

If f is a smooth function on [a,b] then the arc length L of the curve y=f(x) from x=a to x=b will be

► $L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$$L = \int_a^b \sqrt{1+[f'(x)]}dx$$

►

$$L = \int_0^a \sqrt{1+[f'(x)]}dy$$

►

► None of these

Question No: 10 (Marks: 1) - Please choose one

If f is a smooth function on [0,3] then the arc length L of the curve y=f(x) from x=0 to x=3 will be

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}dx$$

►

$$L = \int_a^b \sqrt{1+[f'(x)]^2}$$

►

$$L = \int_0^3 \sqrt{1+[f'(x)]^2}dy$$

►

► None of these

Question No: 11 (Marks: 1) - Please choose one

By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between $y = 3x \ and \ y = 2x^2$ is revolved about the x-axis

$$V = \int_0^{\frac{3}{2}} 2\pi x(3x-2x^2)dx$$

►

$$V = \int_0^{\frac{3}{2}} x(3x - 2x^2)dx$$

►

$$V = \int_0^{\frac{3}{2}} 2\pi(3x - 2x^2)dx$$

►

► None of these

Question No: 12   (Marks: 1)   - Please choose one

By using cylindrical shell to find volume of the solid when the region R in the first quadrant enclosed between $y = x \; and \; y = x^2$ is revolved about the y-axis is represented by

$$V = \int_0^3 2\pi x(x - x^2)dx$$

►

$$V = \int_0^1 x(x - x^2)dx$$

►

$$V = \int_0^1 2\pi(x - x^2)dx$$

►

► None of these

Question No: 13   (Marks: 1)   - Please choose one

If

$$\int_a^a f(x)dx =$$

a is in the domain of f, then

► $f'(x)$

► $f(x)$

► **0**
► None of these

Question No: 14 (Marks: 1) - Please choose one

Consider the integral $\int_0^2 x^2 dx$, the area on right is bounded by

► $y = x^2$
► $x = 2$
► $x = 0$
► None of these

Question No: 15 (Marks: 1) - Please choose one

The series $1-3+5-7+9-11$ may written as in sigma notation

► $\sum_{k=0}^{k=5}(-1)^k(2k+1)$
► $\sum_{k=1}^{k=5}(-1)^k(2k+1)$
► $\sum_{k=1}^{k=5}(2k+1)$
► None of these

Question No: 16 (Marks: 1) - Please choose one

$4^2+5^2+6^2+7^2$ in sigma notation may be represented as

$$\sum_{k=2}^{k=7} k^2$$

► $\sum_{k=2}^{k=7} (k+1)^2$

► $\sum_{k=4}^{k=7} k^2$

►

► None of these

Question No: 17 (Marks: 1) - Please choose one

If a function f is ……….. on a closed interval [a,b] ,then f has both a maximum and minimum value on [a,b]

► **Continuous**
► Discontinuous
► Differentiable
► None of these

Question No: 18 (Marks: 1) - Please choose one

Let f be a function on an interval, and $x_1 \; and \; x_2$ denote the points in that interval, if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$ then the we can say that f is

► Increasing function
► **Decreasing function**
► Constant function
► None of these

Question No: 19 (Marks: 1) - Please choose one

If a function satisfies the conditions

f(c) is defined

$$\lim_{x \to c^+} f(x)$$

Exists

$$\lim_{x \to c^+} f(x) = f(c)$$

Then the function is said to be

- ► **Continuous at c**
- ► Continuous from left at c
- ► Continuous from right at c
- ► None of these

Question No: 20 (Marks: 1) - Please choose one

For a function f(x) to be continuous on interval (a,b) the function must be continuous

- ► **At all point in (a,b)**
- ► Only at a and b
- ► At mid point of a and b
- ► None of these

Question No: 21 (Marks: 2)

$$a_{n+1} = \frac{1}{3}(a_n + \frac{1}{a_n}) \ for \ n \geq 1 \ and \ a_1 = 2$$

Write down the first two term of the sequence

Question No: 22 (Marks: 2)

Find the integral of the surface area of the portion of the sphere generated by revolving the

$$y = \sqrt{2 - x^2}, 0 \leq x \leq \frac{1}{3}$$

curve
(Note: Just find the integral do not solve the integral)

Question No: 23 (Marks: 2)

Calculate $\int_2^5 f(x)dx$ if $\int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$

$$\int_2^3 f(x)dx = 7, \int_3^4 f(x)dx = 2, \int_5^4 f(x)dx = 5$$

$$\int_2^5 f(x)dx = 7 + 2 - 5 = 4$$

Question No: 24 (Marks: 3)

Find the first two Taylor polynomials for $\ln x$ about $x = 3$

Question No: 25 (Marks: 3)

Let the curve $y = x^{\frac{3}{2}}$; $0 \le y \le 2$, then find the surface area generated by revolving the curve. (But do not evaluate)

Question No: 26 (Marks: 3)

Express the sum $\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{7225}$ in sigma notation but do not evaluate.

$$\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + ... + \frac{1}{7225}$$

$$\sum_{k=1}^{7225} k^3 + 1$$

Question No: 27 (Marks: 5)

Find the first four nonzero terms of the Taylor series generated by f at $x = a$

$$f(x) = \frac{1}{1-x} \quad at \quad x = 2$$

Question No: 28 (Marks: 5)

Evaluate the Definite Integral using the First fundamental Theorem of Calculus

$$\int_0^1 (x^5 - x^3 + 2x)dx$$

$$Let\, u = (x^5 - x^3 + 2x)$$

$$\int_0^1 (u)dx$$

Question No: 29 (Marks: 5)

Express the definite integrals as limits (Do not evaluate the integrals)

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + Cosx)\, dx$$

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + Cosx)\, dx$$

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \lim_{\max xk \to 0} \sum_{\frac{-\pi}{2}}^{\frac{\pi}{2}} (1 + \cos x)dx$$

Question No: 30 (Marks: 10)

Find the region enclosed by the curves and also find the area

$$y = x^2 \,,\, y = \sqrt{x} \,,\, x = \frac{1}{4} \,, \quad x = 1$$

Question No: 31 (Marks: 10)

Use x^*_k as the left end point of each subinterval to find the area under

$y = mx \ \ over\, the\, \text{int}\, erval \left[a, b\right], where\, m > 0\, and\ a \geq 0$

Solution on next page

$Suppose \quad a = 1\ b = 2\ so[a, b] = [1, 2]$

$x_k * = x_{k-1} = a + (k-1)\Delta x \qquad (formula\ for\ left\ end\ po\text{int})$

$\Delta x = \dfrac{b-a}{n} = \dfrac{2-1}{n} = \dfrac{1}{n}$

$Suppose\ \ kth\ has\ area$

$f(x_{k*)}\Delta x = x_{k*}\Delta x$

$\left[1 + \dfrac{k}{n}\right]\Delta x$

$\left[1 + \dfrac{k}{n}\right]\dfrac{1}{n}$

$\sum_{k=1}^{n} f(x_{k*})\Delta x = \sum_{k=1}^{n}[1 + k - 1]$

$Area\, by\, solving$

$$A = \lim_{\delta x \to 0}\sum_{k=1}^{n} f(x_{k*})\Delta x = \lim_{\delta x \to 0}\left[\frac{3}{2} - 1 + \frac{1}{2n}\right]$$

$$= \frac{3}{2} - 1 + 0$$